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Accounting for tailings dam failures in the valuation of mining projects

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ABSTRACT

The number of major tailings dam failures has doubled over the past 20 years, culminating in the tragic accident at Brumadinho in Brazil where about 300 people lost their lives. In this context, there is a growing demand from mining companies, institutional investors and policymakers alike for updated mining project assessment tools taking account of such risks. As part of this research, this paper develops a real option framework for evaluating mining projects involving tailings dams and their associated risk. Two options are considered beyond standard business-as-usual safety measures: reinforced dam maintenance, and retrofitting a treatment process that reduces the volume of unconsolidated tailings. A closed-form expression was obtained for the expected value of the business-as-usual case; semi-analytic formulas were obtained for the two options for evaluation by dynamic programming with quantization of the price factor. When applied to an iron ore deposit with characteristics similar to the Samarco deposit, the method shows that both options are financially superior to business-as-usual for the mining company, with the dry processing retrofitting option being the most attractive. The sensitivity of the expected values was evaluated over a range of values of the key parameters. This research provides senior decision-makers with tools to evaluate different options regarding tailings dam safety from a financial point of view, and provides financial evidence in favour of safer treatment processes for mining waste.

1. Introduction

Traditional mining projects store the byproducts of processing operations behind a tailings dam. When the wall of a tailings dam is breached, it sends a flood of often toxic material into the surrounding countryside, causing serious environmental damage and often loss of life. The death toll in the recent disaster at Brumadinho in Brazil was estimated at 300 people (Reuters, 2019). Three years earlier 19 people were killed in the Samarco disaster in November 2015 also in Brazil and the Rio Doce River was polluted for more than 660 km down to its mouth and out into the Atlantic Ocean (Fernandes et al., 2016). One year earlier an ecological disaster occurred in Canada when the tailings dam at the Mt Polley copper and gold mine collapsed, polluting 25 percent of the wild salmon spawning grounds (Amnesty International Canada, 2018).

Tragic consequences are by no means new, especially when the dam is built uphill of a township as was the case in Brumadinho, in Bento Rodrigues and earlier on in Merriespruit (South Africa) where 17 people died in 1994 (Van Niekerk and Viljoen, 2005) and in Aberfan (Wales) where 114 children and 26 adults died in 1966 when the waste from a colliery engulfed a primary school and 20 houses (Couto, 1989).

These repeated incidents raise questions about the viability of the corporations involved and the credibility of the mining industry as a whole as can be seen from the Investor Mining & Tailings Safety Initiative led by The Church of England (2019). In a recent article on the two disasters in Brazil, The Economist (2019) asked whether the fines, lawsuits and damage to Vale's reputation will reach the same level that British Petroleum incurred after the Deepwater Horizon disaster in the Gulf of Mexico in 2010. In that case the total bill for BP came to USD 60bn (The Economist, 2019).

Whereas most papers on this subject focus on the technical reasons for the failures or the environmental damage done, this paper develops a method for evaluating the financial impact of such disasters within a real option framework from the mining company's point of view, taking into account the costs of reparations and fines. Our aim is to provide senior decision-makers (CEOs and board members) with a framework for working out whether the additional cost of replacing the existing mineral processing method with a dry procedure (which does not

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require a tailings dam), or alternatively to significantly increase maintenance, is justified.

The contributions of the paper are threefold. First, we establish a closed-form expression for the expected value of a conventional mining project, subject to the risk of tailings dam failure, accounting for three main stochastic variables: the commodity price, the probability of a tailings dam failure occurring, and the penalty cost. Next, we obtain semi-analytic formulas for the value of the two real options within a simple dynamic programming numerical scheme combined with optimal quantization of the price factor. Finally, we set up a hypothetical iron-ore deposit case-study calibrated based on the available statistical data on tailings dam failures and on the engineering experience of our mining co-authors The key outcome is that once the financial impact of tailings dam disasters is taken into account, it is preferable to retrofit dry processing if that is technically feasible. This brings 20% extra value compared to the base case, while reinforced maintenance adds 8% extra value. These options are not only financially attractive at the outset of the project, but also during the life of existing projects. In conclusion, our paper provides financial evidence in favour of safer treatment processes for mining waste.

The paper is structured as follows. Section 2 reviews the literature on tailings dam failures and real option evaluation of projects, in particular mining ones. In Section 3 we describe the model used to value mining projects involving tailings dams. Three situations are considered: firstly the standard operating procedure, then the option to carry out additional maintenance and finally the option to change the process design to avoid having recourse to tailings dams. The key parameters are described and we explain how their values are chosen. Section 4 presents the numerical results. The conclusions follow in Section 5. Proofs are given in appendix.

2. Literature review

2.1. Tailings dam failures

Although tailings dam failures are often considered extremely rare events, they are far more common than is realized. In the 18 year period from 2000 to mid-2017, 36 such cases occurred, an average of two per year (Wise Uranium, 2018). Most papers on this subject focus on the technical reasons for the failures or the environmental damage done.

2.1.1. Causes of tailings dam failures

After four major accidents had occurred in Europe between 1998 and 2000 (Los Frailes, Spain in 1998; Baia Mare, Romania in January 2000; Baia Borsa, Romania, in March 2000 and Aitik Sweden, in September 2000), the European Union commissioned Rico et al. (2008) to carry out a study. They identified 147 cases worldwide, 26 of which had occurred in Europe. They split the causes of failures into 11 categories, in each case the number of cases in Europe is shown in brackets: foundations (3), slope instability (1), overtopping (2), mine subsidence (2), unusual rainfall (8), snow melt (1), piping/seepage (2), seismic liquefaction (0), structural (1), management or operational error (3), unknown (3). Davies (2002) stressed that many of the failures were due to geotechnical reasons.

The likelihood of failure also depends on whether the dam is sequentially raised by adding material upstream, downstream or along the centerline, with upstream dams being the most dangerous (Kossoff et al., 2014; Martin and McRoberts, 1999). Several days after the Brumadinho disaster, Vale's CEO ordered the decommissioning of all ten of the firm's upstream tailings dams, halting production at the mines nearby. This will affect about 10% of the company's production (The Economist, 2019).

Several authors have highlighted problems specific to Brazil. Labonne (2016) pointed out that as government agencies still lack the capacity to enforce increasingly complex environmental legislation, the industry engages in self-monitoring, which leads it to cutting corners when confronted with an economic downturn. dos Santos and Milanez (2017) reached similar conclusions after studying the relations between the State, the market and civil society and the rearrangement of environmental regulation of the mining industry in Brazil. After studying the Fundão dam collapse which destroyed the Bento Rodrigues township in 2015, do Carmo et al. (2017) concluded that there was an undeniable need for more rigorous control of the hundreds of mining tailings dams in Brazil.

In contrast to most papers which focus on the mechanisms of failure, Bowker and Chambers (2015) came to the conclusion that mining economics plays an important role in tailings dam failures. They highlighted the recent increase in the rate of severe and very serious disasters caused by tailings dam failures and argued that this is a consequence of exploiting lower grade mega-mines which produce far larger quantities of tailings.

2.1.2. Environmental damage caused by these failures

One of the best descriptions of the ecological devastation caused by a tailings dam failure is the World Wildlife Fund report on the damage caused by the collapse of the Los Frailes dam in Spain in 1998 (WWF, 2002b): *The spill flooded the riverbanks along the Agrio and Guadiamar Rivers down to the Entremuros marshes,* 40 km *south of the mine, at the border of the Doñana Natural Park. Land strips 250-m wide on each side of the Guadiamar river were flooded with tailings and toxic water. … During the first few hours after the dam failure, the water in the Agrio and Guadiamar Rivers presented no dissolved oxygen and a very high amount of solids in suspension, which caused the death of all kinds of sub-aqueous life. 30 tons of dead fish and* 170 kg *of dead crabs and amphibians were collected. Adult birds living on the riverbanks could escape the toxic flood but the egg-laying season was severely affected.*

Tailings often contain high levels of heavy metals and other toxic substances (e.g. cyanide). After the Samarco disaster in Brazil, very high levels of Hg, Co, Fe and Ni were found in sediments in the Rio Doce River, and many other heavy metals (Ag, Ba, Cr, Cu, Mn, Pb and Zn) were found in suspended particulate matter (Hatje et al., 2017). According to the authors, the mercury pollution in the river was not from the tailings dam itself. Rather it was due to artisanal gold mining in past centuries but these sediments had been churned up by the flood of tailings coming down the river. On 30 January 2000, the tailings dam at the Baia Mare gold mine in Romania broke, releasing 100,000 m^3 of water containing about 100 tons of cyanide. The water flowed into the Tisza River then into the Danube, causing serious ecological damage in the rivers and on adjacent farmlands (WWF, 2002a).

2.1.3. Reducing the quantity of tailings produced

In the long run, the best way stop these accidents would be to change the processing technology. Vale is using dry processing at its new SIID iron-ore deposit (Vale, 2018; Leahy and Hume, 2016). Other new technologies and innovations, such as thickened tailings, dry stacking and paste backfill, have greatly increased the range of waste disposal methods available to meet the future challenges to sustainable development (Franks et al., 2011). Compared to conventional tailings which contain 30–50% solids, thickened tailings contain 55–75% while paste contains over 75% solids. For example Norsk Hydro has retrofitted filter presses at its Barcarena plant (Norsk Hydro, 2018). Edraki et al. (2014) highlight three approaches: paste and thickened tailings; tailings reuse, recycling and reprocessing; and proactive management (e.g. the integration of sulphide flotation with cemented paste backfill).

2.2. Real options

In our paper, we evaluate a mining project with a tailings dam and the risk of failure from a financial point of view using a real option framework. Financial options are derivative products which give their owner the right (but no obligation) to buy or sell a certain quantity of an asset at a specified price at or before a certain date called the expiry date or maturity (Hull, 2014). The asset could be shares, or a commodity, or an interest rate or an exchange rate. By extension, real options give their owner the right but no obligation to undertake business initiatives that are connected to and exist on or within real assets (e.g. see Amram et al., 1998; Trigeorgis, 1993). The term "real options" was coined by Myers (1977) who divided assets into two categories: assets in place and growth possibilities or real options (Savolainen, 2016). Tourinho et al. (1979), Brennan and Schwartz (1985) and Paddock et al. (1988) were amongst the earliest applications of real options to mining. Brennan and Schwartz (1985) considered a copper mine that in addition to continuing to operate, could stop production temporarily or permanently if the commodity price dropped. Their model gave the cutoff commodity price when it was optimal to switch from operating to temporary or permanent closure, or vice versa. In order to evaluate the project, they set up a replicating self-financing portfolio consisting of the project and the appropriate number of futures; that is, they used the same procedure as for pricing financial options. They assumed that the commodity price followed a Geometric Brownian Motion.

Although the main options considered in real options are stopping and starting the project, deferring expenditures, expanding or contracting the project (Trigeorgis, 1993), many other types of flexibility have been considered. Cortazar and Schwartz (1993) modelled a mine with a stockpile (inventory); Cortazar and Casassus (1998) studied mine expansions; Cortazar et al. (2001) and Langrené et al. (2017) focused on exploration investments under price and geological uncertainties; Moel and Tufano (1999)) used a real options framework to analyse bidding for a mine in an auction organised by the Peruvian government.

Real options have also been applied in many other fields. Dias (2006) used them to evaluate oil projects and Deng and Oren (2006) modelled electricity generation projects. Another important development in real options was the option to learn especially in the R&D sector. Grenadier and Weiss (1997) developed an optimal investment strategy for firms faced with sequential technological innovations while Sadowsky (2005) used real options to value investments in pilot plants. Koussis et al. (2007) focussed on R&D options with "time to learn" and "learn by doing" options.

The real options approaches have evolved to incorporate multiple risk factors and more general dynamics for these risk factors. When a dynamic decision making problem is formulated as a finite horizon discrete stochastic control problem, Monte Carlo regression methods also called Least Squares Monte Carlo, LSMC, can be used to solve it numerically, providing an optimal decision policy (Longstaff and Schwartz, 2001; Abdel Sabour and Poulin, 2006; Tsekrekos et al., 2012). Intuitive decision support tools can be developed to indicate the optimal option for any situation that could happen. Chen et al. (2015) constructed switching boundaries and surfaces for the Brennan and Schwartz (1985) framework using the regression-based Monte Carlo method. Langrené et al. (2015) proposed a non-parametric adaptive local regression method combined with control randomisation and memory reduction techniques to solve realistic high-dimensional real option problems in mining. Chen et al. (2016) implemented LSMC with a control randomisation technique to find the real option value of variable extraction rates in a natural resource extraction problem under a production target constraint.

3. Model description

In this section, we introduce the model used to value of the mine, starting with the uncertain variables, namely the failure time, penalty cost and commodity price. We then compute the annual cashflows, taking into account the tax payment as in Brennan and Schwartz (1985). The value of mine is the total discounted cashflow minus the penalty if a disaster happens. The notations follow Brennan and Schwartz (1985), Chen et al. (2015) and similar papers. After that, we consider the following two possible options for an existing mine with a tailing dam: 1) additional preventive maintenance and 2) dry processing retrofit, and then explain the procedure for calibrating the parameter values.

3.1. The model

The three key sources of uncertainty in this model are the time at which the tailings dam failure occurs if that happens, the subsequent penalty and cost to the mining companies and the commodity price. For simplicity, we assume that the mine has a fixed finite lifespan $T > 0$, and that the reserves are large enough for the mine to keep producing at the rate of q during that time. The commodity price $S = (S_t)_{0 \le t \le T}$ is assumed to follow a mean-reverting positive process. A Weibull distribution is used to model the disaster occurrence time τ. In the event of a disaster, the mine has to shutdown for a certain time and the company has to pay fines and for the reparation of the damage. A log-normal distribution is used to model the penalty cost P_τ to the mining company. Its mean and variances both increase with time. For simplicity, the length of shutdown and recovery period D are assumed to be constant.

3.1.1. Distribution for failure time

In survival analysis, it is natural to model the time to failure by a Weibull distribution, which is a powered exponential distribution (Mann et al., 1974). We model the failure time τ as a Weibull random variable with rate $\lambda > 0$ and shape parameter $k > 1$. More specifically, the probability distribution function f_τ of τ is given by

$$f_\tau(t) = \lambda k t^{k-1} e^{-\lambda t^k} 1\{t \ge 0\} \tag{1}$$

where the $1\{t \ge 0\}$ is the indicator function which equals 1 if $t \ge 0$ and 0 if otherwise. The corresponding failure (or hazard) rate $h(t)$ representing the frequency with which the dam fails is given by

$$h(t) = \frac{f_\tau(t)}{1 - F_\tau(t)} = \lambda k t^{k-1}$$

for any $t \ge 0$, where $F_\tau(t) = 1 - e^{-\lambda t^k}$ is the cumulative failure distribution function of τ (Papoulis and Pillai (2002)). In the case where $k = 1$, the hazard rate is equal to the constant λ, and the distribution (1) coincides with an exponential distribution. In the case where $k > 1$ the hazard rate increases over time; for example, in the case $k = 2$ (equivalent to a Rayleigh distribution), the hazard rate grows linearly over time. The failure of a tailings dam can be viewed as an "*aging*" process that is more likely to fail as time goes on, because the longer the mine has been in operation, the more waste is stored behind the dam. Thus there is higher chance of dam failure. To account for this increasing likelihood of failure, we make the conservative assumption that $k = 2$ (linear growth of failure rate). We also assume the failure time τ, the penalty cost in the event of a disaster, the shut down period after a disaster and the commodity price are independent of each other. Appendix A provides useful formulas regarding τ.

3.2. Cost of failure and duration of shutdown

If a tailings dam failure occurs, the company has to pay a penalty cost $P_\tau > 0$ to cover the repairs and any fines incurred. To account for the uncertainties surrounding such fines, we model the penalty cost as a lognormal variable with mean $\mu_P(t) > 0$ and standard deviation $\sigma_P(t) > 0$. The analytical formula for the penalty cost at time t is $P_t = e^{\mu_P(t) + \sigma_P(t) N}$, where N is a standard Gaussian variable. In view of the heterogeneity in the few available penalty cost estimates following tailings dams disasters (Davies (2002); Azam and Li (2010)), we found that a simple lognormal distribution (nonnegative, heavy-tailed) was a suitable a priori choice for the penalty distribution. We assume that both $\mu_P(t)$ and $\sigma_P^2(t)$ grow linearly with time: $\mu_P(t) = \mu_P t$ and $\sigma_P^2(t) = \sigma_P^2 t$ where $\mu_P > 0$ and $\sigma_P > 0$. If we let $p: =\mu_P + \frac{1}{2}\sigma_P^2$, then the expected penalty cost $\mathbb{E}[P_t] = e^{pt}$ grows exponentially with time t.

After a failure, the mine is forced to shut down for a recovery and reconstruction period $D > 0$. This period depends on many factors such as disaster severity, location, length of investigation and legal process etc. After a disaster, some mines do reopen with restricted operations, or may even go back to full production after disaster recovery and reconstruction. For example, the Mount Polley mine reopened 2 years after the tailings pond disaster in 2014. In other cases the mine might never reopen.

3.3. Commodity price

We model the commodity price (iron ore) by the following mean-reverting positive process:

$$dS_t = \kappa_S(\theta_S - S_t)dt + \sigma_S S_t dW_t \tag{2}$$

where $W = (W_t)_{0\le t\le T}$ is a standard Brownian motion, κ_S is the mean-reversion speed, θ_S is the mean-reverting level, and $\sigma_S > 0$ is the volatility. This simple one-factor model, known as mean-reverting GBM, or Inverse Gamma process (Zhao (2009); Langrené et al. (2016)), accounts for the long-term mean-reversion common to many commodity prices (Schwartz (1997); Andersson (2007)) including metal prices. Mean-reverting models are commonly used for metal prices, including iron (Cortazar and Casassus (1998); Bernard et al. (2008); Ajak et al. (2018)). While several possible mean-reverting one-factor stochastic models exist in the literature (Kloeden and Platen (1992, Chap. 4), Aba Oud and Goard (2015)), we find the simple linear mean-reversion of equation (2) easier to interpret. The expectation of S_u, $u \ge t$, given S_t is

$$\mathbb{E}[S_u|S_t] = \theta_S + (S_t - \theta_S)e^{-\kappa_S(u-t)} \tag{3}$$

3.4. Taxes

Let $\text{Tax}(S_t, C_t)$ denote the total income tax and royalties. This depends on the price S_t and cost C_t and the production rate q. As in Brennan and Schwartz (1985) and similar papers, we assume that

$$\text{Tax}(S_t, C_t) = p_1 q S_t + p_2 q(S_t(1 - p_1) - C_t)$$

where p_1 and p_2 are the royalty rate and the corporate tax respectively, so that the instantaneous after tax cash-flow at time t is the profit minus tax as follows:

$$\begin{aligned}\Pi(t_t, S_t) &= q(S_t - C_t) - \text{Tax}(S_t, C_t)\\ &= qw_S S_t - qw_C C_t\end{aligned} \tag{4}$$

where $w_S: =1 - p_1 - p_2 + p_1p_2$ and $w_C: =1 - p_2$.

3.5. Value of mine: discounted cash flow

Having introduced the components, we can now define the value of mine. The cash flow between time t and the end of time horizon T is the accumulated discounted cashflow after paying tax and after deducting the penalty cost if a disaster happens. The total discounted cash flow $\text{CF}_{t,T}$ during the time interval $[t, T]$, with $0 \le t \le T$ can be written as:

$$\text{CF}_{t,T} = \int_t^{\tau\wedge T} e^{-ru}\Pi(u, S_u)du - e^{-r\tau}P_\tau 1\{\tau \le T\} + \int_{(\tau+D)\wedge T}^{T} e^{-ru}\Pi(u, S_u)du \tag{5}$$

where $a \wedge b = \min\{a, b\}$. In this context, it means the time a or b, whichever happens first.

Eq. (5) involves three terms. The first term and the last term correspond to the discounted cash-flows between two time points: t and until a failure time τ and/or end of licence T, whichever comes first and the post-recovery period $\tau + D$ if the mine can recover before the licence expires at time T. The middle term $- e^{-r\tau}P_\tau$ corresponds to the expected discounted penalty cost for a disaster at time τ, and the factor $1\{\tau \le T\}$ is 1 if $\tau \le T$, that is, if there is a disaster at the time τ during the life of the mine. The corporate discount rate is r.

3.6. Mine option value

This section establishes an explicit formula for the expected value of the mine (business-as-usual without exercising any option), as well as explicit inductions for the other two options.

3.6.1. Base case: business as usual

The expected value of the mine at time $t_0 = 0$ is given by

$$\begin{aligned}&v_{0,T}(0, S_0) = \mathbb{E}[\text{CF}_{0,T}]\\ &= \mathbb{E}\left[\int_0^{\tau\wedge T} e^{-ru}\Pi(u, S_u)du - e^{-r\tau}P_\tau 1\{\tau \le T\} + \int_{(\tau+D)\wedge T}^{T} e^{-ru}\Pi(u, S_u)du\right]\end{aligned} \tag{6}$$

Theorem 1. *Under the assumptions from Subsection 3.1, the expected value (6) of the mining project at the initial time is given by*

$$\begin{aligned}&v_{0,T}(0, S_{t_0})\\ &= qw_S\theta_S(\boldsymbol{\Psi}_{0,T}(r, 0) + \xi_{0,T}(r) - \boldsymbol{\Psi}_{0,T}(r, D))\\ &+ qw_S(S_0 - \theta_S)(\boldsymbol{\Psi}_{0,T}(r + \kappa_S, 0) + \xi_{0,T}(r + \kappa_S) - \boldsymbol{\Psi}_{0,T}(r + \kappa_S, D))\\ &- qw_C C_0(\boldsymbol{\Psi}_{0,T}(r - \pi, 0) + \xi_{t,T}(r - \pi) - \boldsymbol{\Psi}_{t,T}(r - \pi, D)) - \Theta_{t,T}(r - p)\end{aligned} \tag{7}$$

where $\xi_{0,T}(r)$ is given by equation (19) *(with $t = 0$), $\Theta_{0,T}(r)$ is given by* equation (17) *(with $t = 0$), and $\boldsymbol{\Psi}_{t,T}(r, D)$ is given by* equation (23) *(with $t = 0$). The definitions of w_S and w_C were given in* section 3.1.3.

Proof. Take $t = 0$ and $s = \infty$ in equation (28) (Proposition 7).

3.6.2. Option 1: preventive maintenance option

We now consider the option to perform maintenance on the dam (reinforcing or heightening the dam for example) whenever the risk is deemed too high. Let C_{maint} stand for the total upfront cost of such maintenance, and assume the maintenance causes a forced shutdown of fixed duration D_{maint}, during which the production is stopped and no tailings dam failure can occur. After the maintenance, we assume the risk of a tailings dam failure is decreased from a rate λ to a lower rate $\lambda_{\text{maint}} \in (0, \lambda)$. We assume the option to perform such preventive maintenance can be made at the fixed decision times $t_0 = 0 < t_1 < t_2 < \ldots < t_N = T$, but it cannot be made if a tailings dam disaster has already occurred, and can only be made once. Let $v^{\text{maint}}(t_n, S_{t_n})$ denote the expected value of the mine at time t_n and metal price S_{t_n} when such a maintenance option is available, conditionally on no tailings dam disaster before t_n. This value function v^{maint} satisfies the following dynamic programming principle:

$$\begin{aligned}&v^{\text{maint}}(T, S_T) = 0\\ &v^{\text{maint}}(t_n, S_{t_n}) = \max\left\{-e^{-rt_n}C_{\text{maint}} + \mathbb{E}\left[\text{CF}^{\lambda\text{maint}}_{(t_n+D_{\text{maint}})\wedge T,T}\middle|S_{t_n}, \tau^{\lambda\text{maint}}_{t_n} > (t_n + D_{\text{maint}}) \wedge T\right],\right.\\ &\mathbb{E}[\text{CF}_{t_n,T}|S_{t_n}, t_n \le \tau_{t_n} < t_{n+1}]\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})\\ &\left.+ (\mathbb{E}[\text{CF}_{t_n,t_{n+1}}|S_{t_n}, \tau_{t_n} > t_{n+1}] + \mathbb{E}[v^{\text{maint}}(t_{n+1}, S_{t_{n+1}})|S_{t_n}])\mathbb{P}(\tau_{t_n} \ge t_{n+1})\right\},\quad t_n < T\end{aligned} \tag{8}$$

- At time T, which coincides with the end of the lease, there is no cash-flow left and the value is zero.
- At time $t_n < T$, the manager has the option to decide to perform preventive maintenance on the dam
  - If the decision is to perform such maintenance, the expected value of the mine (discounted to time zero) is given by the cost of

Table 1
Table of parameters.

Variables		Value	Variables		Value
T	mine life	27 years	S_0	initial price	70.58 USD/ton
D	disaster shutdown	5 years	θ_S	price mean	77.90 USD/ton
D_{maint}	maintenance shutdown	1 year	κ_S	mean-rev. speed	0.65
λ	parameter of failure rate	2.4×10^{-4}	σ_S	volatility	0.26
q	production rate per year	25 million ton	p_1	royalty rate	2%
p	penalty rate	0.15	p_2	tax rate	34%
r	discount rate	4%			

maintenance $-\, e^{-rt_n} C_{\text{maint}}$ plus the expected cash-flows after the maintenance shutdown, conditionally on the metal price S_{t_n} at the current time $\mathbb{E}[\text{CF}^{\lambda_{\text{maint}}}_{(t_n+D_{\text{maint}})\wedge T,T}|S_{t_n}, \tau^{\lambda_{\text{maint}}}_{t_n} > (t_n + D_{\text{maint}}) \wedge T]$. (We assume that the dam cannot fail during the shutdown).

– If the decision is not to perform such maintenance and to wait for the next decision time, the expected value of the mine (discounted to time zero) is given by:
  * If a tailings dam disaster occurs during the time period $[t_n, t_{n+1})$ (with probability $\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})$), the value is given by the passive cash-flows $\mathbb{E}[\text{CF}_{t_n,T}|S_{t_n}, t_n \le \tau_{t_n} < t_{n+1}]$ (as the preventive maintenance decision is no longer available after the disaster).
  * If no tailings dam disaster occurs during the time period $[t_n, t_{n+1})$ (with probability $\mathbb{P}(\tau_{t_n} \ge t_{n+1})$), the value is given by the sum of the cash-flows on the time period $[t_n, t_{n+1})$, given by $\mathbb{E}[\text{CF}_{t_n,t_{n+1}}|S_{t_n}, \tau_{t_n} > t_{n+1}]$, and the expected future cash-flows, given by $\mathbb{E}[v^{\text{maint}}(t_{n+1}, S_{t_{n+1}})|S_{t_n}]$ (as the preventive maintenance option remains available for the future decision times).

Using the explicit formulas from Appendix B, equation (8) can be written as:

$$v^{\text{maint}}(T, S_T) = 0$$
$$v^{\text{maint}}(t_n, S_{t_n}) = \max\Big\{-e^{-rt_n}C_{\text{maint}} + v^{\lambda_{\text{maint}}}_{(t_n+D_{\text{maint}})\wedge T,T}(t_n, S_{t_n}),$$
$$v_{t_n,T}(t_n, S_{t_n}, t_{n+1})\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})$$
$$+ (v^{\text{safe}}_{t_n,t_{n+1}}(t_n, S_{t_n}) + \mathbb{E}[v^{\text{maint}}(t_{n+1}, S_{t_{n+1}})|S_{t_n}])\mathbb{P}(\tau_{t_n} \ge t_{n+1})\Big\}, \quad t_n < T \tag{9}$$

where $v_{t,T}(t_0, S_{t_0}, s)$ is given by equation (28), $v^{\text{safe}}_{t,T}(t_0, S_{t_0})$ is given by equation (27), $\mathbb{P}(t \le \tau_t < s)$ is given by equation (12) and $\mathbb{P}(\tau_t \ge s)$ is given by equation (13). In other words, all the terms in equation (9) are explicit except for $\mathbb{E}[v^{\text{maint}}(t_{n+1}, S_{t_{n+1}})S_{t_n}|]$, so that the mine value $v^{\text{maint}}(t_n, S_{t_n})$ can be conveniently estimated by induction, as is done in Section 4.

3.6.3. Option 2: dry processing retrofitting option

Finally, we now consider the option to retrofit an alternative design with no tailings ponds (dry processing). Let C_{dry} stand for the total upfront cost of such retrofitting, which is much higher than C_{maint} and possibly higher that the initial cost of building the mine. This retrofitting incurs a forced shutdown of fixed duration D_{dry} (longer than D_{maint}), during which the production is stopped and the tailings pond is progressively decomissioned. After the retrofitting, the risk of a tailings dam failure is effectively removed ($\lambda_{\text{dry}} = 0$). Again, we assume that the option to perform such a retrofitting can be made at the fixed decision times $t_0 = 0 < t_1 < t_2 < \dots < t_N = T$, cannot be made if a tailings dam disaster has already occurred, and can only be made once. Let $v^{\text{dry}}(t_n, S_{t_n})$ denote the expected value of the mine at time t_n and metal price S_{t_n} when such a retrofitting option is available, conditionally on no tailings dam disaster before t_n. This value function v^{dry} satisfies the following dynamic programming principle:

$$v^{\text{dry}}(T, S_T) = 0$$
$$v^{\text{dry}}(t_n, S_{t_n}) = \max\Big\{-e^{-rt_n}C_{\text{dry}} + \mathbb{E}\Big[\text{CF}^{\lambda_{\text{dry}}}_{(t_n+D_{\text{dry}})\wedge T,T}|S_{t_n}\Big],$$
$$\mathbb{E}[\text{CF}_{t_n,T}|S_{t_n}, t_n \le \tau_{t_n} < t_{n+1}]\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})$$
$$+ (\mathbb{E}[\text{CF}_{t_n,t_{n+1}}|S_{t_n}, \tau_{t_n} > t_{n+1}] + \mathbb{E}[v^{\text{dry}}(t_{n+1}, S_{t_{n+1}})|S_{t_n}])\mathbb{P}(\tau_{t_n} \ge t_{n+1})\Big\}, \quad t_n < T \tag{10}$$

The decomposition is similar to the one for the maintenance case (equation (8)), except that $\lambda_{\text{dry}} = 0$, which makes the cash-flows after retrofitting safe. Using the explicit formulas from Appendix B, equation (10) can be written as:

$$v^{\text{dry}}(T, S_T) = 0$$
$$v^{\text{dry}}(t_n, S_{t_n}) = \max\Big\{-e^{-rt_n}C_{\text{dry}} + v^{\text{safe}}_{(t_n+D_{\text{dry}})\wedge T,T}(t_n, S_{t_n}),$$
$$v_{t_n,T}(t_n, S_{t_n}, t_{n+1})\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})$$
$$+ (v^{\text{safe}}_{t_n,t_{n+1}}(t_n, S_{t_n}) + \mathbb{E}[v^{\text{dry}}(t_{n+1}, S_{t_{n+1}})|S_{t_n}])\mathbb{P}(\tau_{t_n} \ge t_{n+1})\Big\}, \quad t_n < T \tag{11}$$

where once again $v_{t,T}(t_0, S_{t_0}, s)$ is given by equation (28), $v^{\text{safe}}_{t,T}(t_0, S_{t_0})$ is given by equation (27), $\mathbb{P}(t \le \tau_t < s)$ is given by equation (12) and $\mathbb{P}(\tau_t \ge s)$ is given by equation (13).

3.7. Calibration

As this paper was written after the Samarco disaster at Bento Rodrigues but before the one at Brumadinho, we constructed the example to approximately fit the first of these mines. Table 1 shows the numerical values that were used for parameters in the example.

3.7.1. Mine life T and production rate q

This can vary from about 5 to 10 years (for small deposits like satellite deposits) out to 60–70 years (Codelco's copper mines). Using Taylor's Law (Devon Smith, 2016), we obtained a mine life of 27 years and a concentrate production rate of 25 million tonnes per year for a mine with 2900 million tonnes of reserves like Samarco. Taylor's law is an empirical result obtained by studying producing mines, which says that

$$\text{Mine Life} = 0.2 \times (\text{Reserves})^{0.25}$$

$$\text{Production Rate} = 0.143 \times (\text{Reserves})^{0.75}$$

where the Reserves are in metric tonnes, the Mine Life is in years and the Production Rate is in tonnes per day on the basis of 350 working days per year. Here "Expected reserves" are generally interpreted to mean proven + probable reserves.

3.7.2. Failure rate for tailings dams

In order to model the impact of tailings dam failures we need an estimate of how likely it is for a dam to fail. Two such estimates are available in the literature. Davies (2002) based his estimate on the assumption that there were about 3500 tailings dams worldwide, of which 2 to 5 failed per year. This equates to an annual probability somewhere between 1 in 700 to 1 in 1750, with an average of

$0.5*\frac{1}{700} + 0.5*\frac{1}{1750} = 0.001$. According to Azam and Li (2010) the failure rate for tailings dams is 1.2% per annum compared to about 0.01% for conventional water retention dams (ICOLD, 2001). We chose to set the failure rate to the average of these two estimates 0.012 and 0.001, namely 0.0065.Within model (1), the average failure rate is equal to $\frac{1-e^{-\lambda T^2}}{T} \simeq \lambda T$. So the value of λ was set to $\lambda = \frac{\mathbf{0.0065}}{\mathbf{T}} \simeq \mathbf{2.4 \times 10^{-4}}$.

3.7.3. Penalty cost (μ_P and σ_P) and shutdown period D

According to the Canadian Miner Martine (2018), two figures have been suggested for the penalty costs for Samarco: \$5B & \$55B. In our model, the penalty cost at year t is equal to $P_t = e^{\mu_P t + \sigma_P t N}$ where N is a standard Gaussian variable. We chose to set the 1% and 99% quantiles of P_{T_1} to \$5B and \$55B respectively, where $T_1 \in [0, T]$. This gives

$$\mu_P T_1 = \frac{\ln(5) + \ln(55)}{2}, \qquad \sigma_P T_1 = \frac{\ln(5) - \ln(55)}{2\mathcal{N}^{-1}(0.01)}$$

where $\mathcal{N}^{-1}$ is the inverse cumulative distribution function of a standard Gaussian variable. In particular, $\mathbb{E}[P_t] = e^{pt}$ where

$$p = \mu_P + \frac{1}{2}\sigma_P^2 = \frac{1}{T_1}\frac{\ln(5) + \ln(55)}{2} + \frac{1}{2T_1^2}\left(\frac{\ln(5) - \ln(55)}{2\mathcal{N}^{-1}(0.01)}\right)^2$$

This approach is illustrated in Fig. 1. A natural value for T_1 is the expected year of a tailings dam disaster conditional on such a disaster happening before the end of the life of the mine. This is given explicitly by equation (14) in the appendix (with $t = 0$ and $s = T$) and yields $T_1 \simeq 17.7$. This choice yields $p \simeq 0.157$ (with the penalty expressed in billions of dollars).

An alternative choice for p is to assume that the higher penalty value \$55B is the expected penalty at the end of life of the mine (at time T, when the expected penalty is the highest). That choice yields $p \simeq 0.148$. As a conservative estimate between these two values, we set $\mathbf{p = 0.15}$ as our base case. Fig. 2 shows the expected penalty value over time for this choice of p. We use a fixed shutdown period of five years $D = 5$ for our base case.

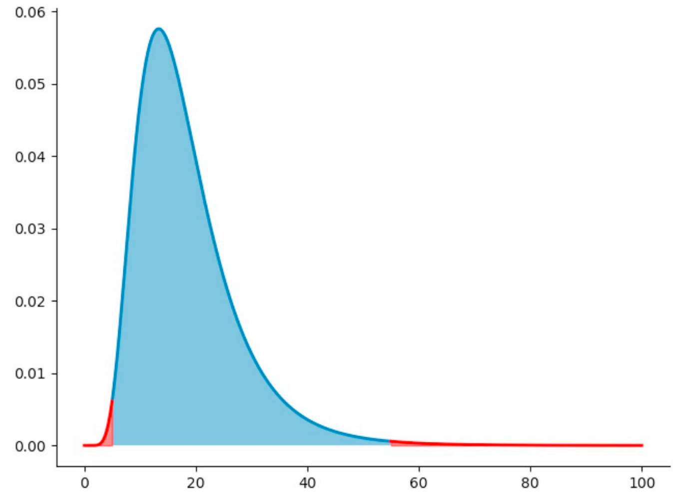

Fig. 1. Fitting penalty distribution.

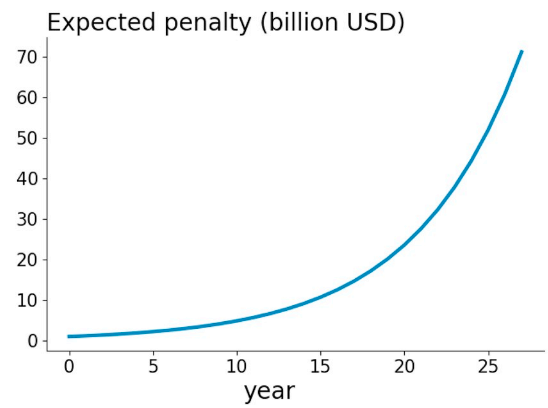

Fig. 2. Expected penalty over time.

3.7.4. Royalty rate p_1 , tax rate p_2 and discount rate r

In Brennan and Schwartz (1985), the royalty rate is 10%, compared to 2% for iron ore in Brazil [1] and to 6.5% [2] in Western Australia in 2016. Brennan and Schwartz (1985) used $p_2 = 50\%$ for corporate tax rate. The current corporate rates in Brazil and Australia are 34% and 30% respectively .[3]

Brennan and Schwartz (1985) set the riskfree rate to $r = 10\%$, which is high compared to current interest rates. The corporate discount rate should equal the WACC which depends on the bond rate and the equity borrowing rate.

According to the BHP report in Aug 2, 017,[4] its USD bonds were paying 2.875%, 3.25% and 3.85% compared to a US Treasury rate of 1.875%. According to the Telegraph, the dividend yield was 4.56% and the debt to equity ratio was 0.73 (3 parts debt to 1 part equity). Consequently

$$\text{WACC}_1 = (3 \times 3.25\,\% + 1 \times 4.56\%)/4 = 3.577\%$$

$$\text{WACC}_2 = (3 \times 3.85\,\% + 1 \times 4.56\%)/4 = 4.045\%$$

Thus in our base model, we adopted $r = 4\%$.

3.7.5. Commodity price parameters

As we are considering an iron ore mine we calibrated the price model (2) on the historical iron ore prices reported in Table 2. The data covers the time period 2011–2018, and prices are expressed in US\$/ton.

We obtained the following estimates for the three parameters of the process in Eq (2):

$\kappa_S = \mathbf{0.65}$ for the mean-reverting speed, $\sigma_S = \mathbf{0.26}$ for the price volatility and $\theta_S = \mathbf{77.90}$ for the mean-reversion price level. We also set $\mathbf{S_0 = 70.58}$ for the initial price.

3.7.6. Opex and Capex

The "six-tenths" rule (de la Vergne, 2003) is widely used to obtain preliminary estimates of mining costs. When the cost of a plant (A) with a certain capacity is known, the cost for another plant (B) can be found using the following rule of thumb:

$$\text{Cost(B)} = \text{Cost(A)} \times (\text{Capacity(B)}/\text{Capacity(A)})^{0.6}$$

Carneiro and Fourie (2018) provide the capital costs for tailings dams with a capacity of 2 Mtpy for various tailings disposal systems while the total Opex for the options was obtained from a Brazilian iron mine's income statement for 2015. The six-tenths rule was then used to estimate the Opex and Capex appropriate to our case-study. The operating costs per ton for the base case and the two options are: Opex0 = US\$ 49.6, Opex1 = US\$ 50.8, and Opex2 = US\$ 51.5 while the capital expenditures in billion US\$ are: Capex0 = 0.035, Capex1 = 0.046, Capex2 = 0.107.

[1] https://www.reuters.com/article/us-brazil-mining-royalties-idUSKBN15A2S6.
[2] https://www.jtsi.wa.gov.au/docs/default-source/default-document-library/wa-iron-ore-profile—august-2017.pdf?sfvrsn=2.
[3] https://home.kpmg.com/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online/corporate-tax-rates-table.html.
[4] https://www.bhp.com/-/media/documents/media/news/2017/170822_bhplaunchesusbondrepurchaseplan.pdf?la=en.

Table 2
Iron prices (Bloomberg ISIX62IU Index).

date	30/12/11	31/12/12	31/12/13	31/12/14	31/12/15	30/12/16	29/12/17	31/12/18
price	138.2	140.9	133.41	69.3	43.4	78.06	70.78	70.58

4. Numerical results

This section presents our numerical results. We start by comparing the mine's value in the three different situations:

- The business-as-usual case (i.e. Base Case) corresponds to a traditional mine without considering safety options such as preventive maintenance or dry processing retrofitting. The value of the mine in this case is obtained by computing equation (6) using the operational costs opex0 and subtracting the capital expenditure capex0 to the result. The expected value of the mine in this case is $\mathbf{V_0 = \$5.41B}$.
- The preventive maintenance case (i.e. Option 1) corresponds to a traditional mine with enhanced maintenance put in place at the outset. The value of the mine in this case is obtained by computing (6) using the higher operational cost opex1 and subtracting the higher capital expenditure capex1 from the result. In addition, to account for the lower risk of tailings dam failure because of the enhanced maintenance, we assumed that the risk of failure is halved, i.e. $\lambda/2$. The expected value of the mine in this case is $\mathbf{V_0^{maint} = \$5.85B}$ (+8%).
- The dry processing case (i.e. Option 2) corresponds to a mine designed at the outset with a dry processing technique. The value of the mine in this case is obtained by computing (27) using the higher operational costs opex2 and subtracting the higher capital expenditure capex2 from the result. In particular, we assume that dry processing mines have no risk of tailings dam failure. The expected value of the mine in this case is $\mathbf{V_0^{dry} = \$6.49B}$ (+20%).

This is one of the main empirical findings of the paper: with the calibrated parameters from subsection 3.3, a dry processing mine has a higher expected value (+20%) than a traditional mine, once the cost of tailings dam failure is properly accounted for. The expected value of a traditional mine with enhanced dam maintenance lies between the two (+8% compared to traditional mine). The two parameters out of those in subsection 3.3 that have the most effect on the numerical results are the probability and severity of tailings dam failures, namely λ and p. Figs. 3 and 4 present the expected mine value in the three scenarios described above, as a function of λ and p respectively.

Fig. 3 shows that dry processing yields a higher value than traditional mines, with and without enhanced maintenance, as long as $\lambda \geq \lambda^* \simeq 8 \times 10^{-5}$ (corresponding approximately to an average yearly failure rate greater than 0.22%, recalling the relation between yearly failure rate and λ from subsection 3.3). Our best estimate for λ (subsection 3.3) was 2.4×10^{-4}, which is three times greater than the limit value λ^* (the x-axis of Fig. 3 is on a log-scale). This gives some confidence to our empirical ranking Base Case < Option 1 < Option 2. Having said that, it is important to acknowledge that the estimated range for λ in Davies (2002), namely $[2.1 \times 10^{-5}, 5.3 \times 10^{-5}]$ is below λ^*. However, the value given in the more recent study Azam and Li (2010) was equivalent to $\lambda = 4.4 \times 10^{-4}$, which is more than five times greater than λ^*. Moreover, the difference between the three expected mine values does not exceed 8% in the region $\lambda < \lambda^*$, whereas the region $\lambda > \lambda^*$ leads to more dramatic differences (+ 21% for Option 1 and + 50% for Option 2 compared to the Base Case with the failure rate estimated from Azam and Li (2010)). This severity imbalance between the two regions suggests that our estimates of + 8% for Option 1 and + 20% for Option 2 might be considered conservative estimates.

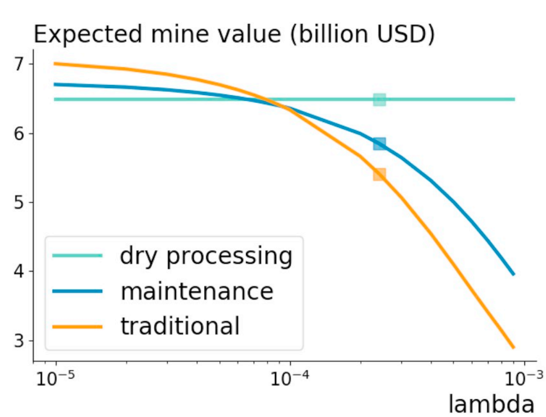

Fig. 3. Expected value w.r.t. λ (log-scale).

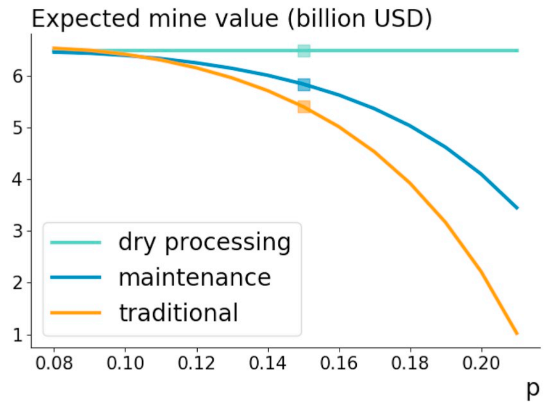

Fig. 4. Expected value w.r.t. p.

Fig. 4 shows that dry processing yields a higher value than traditional mines as long as $p \geq p^* \simeq 0.09$ (corresponding to about an average penalty of \$4.9B at time T_1. (The relation between expected penalty at time T_1 and p is given in subsection 3.3). This is almost three times less than our best estimate of \$14.2B ($p = 0.15$, see subsection 3.3) which again gives some confidence to our empirical ranking Base Case < Option 1 < Option 2.

The results obtained so far apply to new mining projects at their onset. To complement these results, it is interesting to look at the possible decisions available for mining companies regarding existing mining projects. Following subsection 3.2, we focus on the options to undergo preventive maintenance and to retrofit the mine to dry processing. Equations (9) and (11) describe the explicit dynamic programs which provide the value of the mining project when these options are available. We assume that the cost of exercising these options is also equal to capex1 and capex2 respectively.

In order to solve equations (9) and (11) numerically, we use the algorithm based on quantization and interpolation described in Balata et al. (2019), because Bachouch et al. (2019) showed that it performed very well for univariate problems and was competitive against alternative deep learning approaches. For comprehensiveness, the details about the quantization-and-interpolation algorithm are provided in

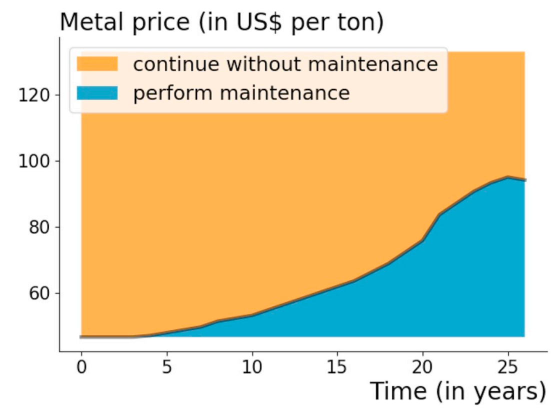

Fig. 5. Maintenance timing.

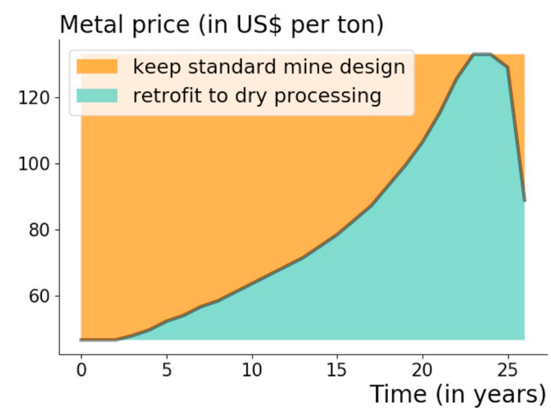

Fig. 6. Dry processing retrofitting timing.

- On Fig. 5, the area where it is optimal to undergo preventive maintenance (provided it has not been done yet) is displayed in blue, while in the orange area it is better not to carry out additional maintenance. One can see that the lower the price, the sooner preventive maintenance should be implemented. One interpretation is that high prices can cover the expected losses incurred when running the mine in an unsafe, unmaintained state. Conversely, safety is critical for profitability in a low-price environment. This brings the best time to perform preventive maintenance forward.
- On Fig. 6 the area where it is optimal to retrofit the mine to dry processing (provided it has not been done yet) is displayed in green, while the "do nothing" zone is shown in orange. Once again, safety is critical for profitability in a low-price environment, in which case the retrofitting is performed earlier. The optimal retrofit region is greater than the optimal maintenance region, so much so that it is eventually optimal to retrofit all mining projects to dry processing before their end, even in very high-price situations as compared to our initial price of $70.58/ton.

Appendix D.

In our tests, this algorithm performed very well on the two univariate real options problems (9) and (11), and produced accurate and stable results which are displayed on Figs. 5 and 6. These two figures show the optimal exercise decisions as a function of the age of the mine (between 0 and $T = 27$ years) and of the commodity price (the price range shown has been chosen to cover the central 99% of the stationary commodity price distribution, see Appendix D.2).

These figures provide a financial guideline for mining companies regarding the costs and benefits of undergoing dam maintenance or dry processing retrofitting over time. It can also help the regulator to identify the dams that are less likely to be spontaneously enhanced by their owner from a cost-benefit point of view.

5. Discussion and conclusions

Soon after the catastrophic tailings dam failure at Brumadinho, Minas Gerias, Brazil, which led to massive loss of life, a group of 96 institutional investors (led by the Church of England Pension Fund and representing more than $10.3 trillion assets under management) set up the Investor Mining and Tailings Safety Initiative. This illustrates the extent of shock worldwide caused by recurring tailings dam failures. Mining companies will effectively need to consider changing the processing procedures or at the very least implementing reinforced maintenance on tailings dams. In doing so, decision-makers are going to need a method for evaluating the costs of different options against the potential costs of reparations and fines.

This paper proposes a real option framework for evaluating two types of options compared to the business-as-usual base case. In the first option (reinforced maintenance) the probability of a dam failure is significantly reduced but not eliminated; the second option (dry processing) eliminates that risk completely. Companies will be able to envisage various options for reducing the risk, which will fit into either the first or second categories of the proposed framework.

We first established a closed-form expression for the expected value of a conventional mining project (i.e. business-as-usual) subject to the risk of tailings dam failure that takes account of the three main stochastic variables: the commodity price, the penalty cost and the probability of a tailings dam failure occurring. Next we obtained semi-analytic formulas for the value of the two real options by a simple dynamic programming numerical scheme combined with Monte Carlo simulations of the dynamic risk factors. This allows mining companies to analyse the attractiveness of these two options.

A case-study on a hypothetical iron-ore deposit was presented. The parameter values were calibrated based on the available statistical data on tailings dam failures and on the engineering experience of our mining co-authors. The key result out of this case-study is that once the financial impact of such disasters is taken into account, it is preferable to retrofit dry processing if that is technically feasible. The value of the project becomes 20% higher than for the base case, while the increased maintenance option leads to an 8% improvement in value. So both options are financially attractive options for mining projects involving tailings dams. These options are not only attractive at the outset of a mining project, but also during the life of an existing project.

Finally our findings could also be used by regulators, NGOs and investors to push for improved safety. In addition to the financial benefits for the companies themselves established in this paper, the case for moving to safer treatment process with less unconsolidated tailings becomes overwhelming when taking heed of the self-evident social and environmental benefits.

Future work could include the joint analysis of the two options (as opposed to one option at a time), the comparison of several types of dam reinforcements, the analysis of alternative stochastic models, and further research to improve the estimates of disaster frequency.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.resourpol.2019.101461.

Appendix

A. Disaster time formulas

Recall from Section 3 that $\tau = \tau^{(\lambda)}$ denotes the stopping time modelling the arrival of a tailings dam disaster. Its probability density function $f_\tau = f_\tau^{(\lambda)}$ is given by $f_\tau(u) = 2\lambda u e^{-\lambda u^2} 1\{u \geq 0\}$. More generally, the probability density function of the conditional disaster times τ_t: $=\tau|\{\tau > t\}$ and $\tau_{t,s}$: $=\tau|\{t \leq \tau < s\}$ are respectively given by

$$f_{\tau_t}(u) = 2\lambda u e^{-\lambda(u^2 - t^2)} 1\{u \geq t\}$$

$$f_{\tau_{t,s}}(u) = 2\lambda u e^{-\lambda u^2} / (e^{-\lambda t^2} - e^{-\lambda s^2}) 1\{t \leq u < s\}$$

The following formulas hold:

$$\mathbb{P}(t \leq \tau_t < s) = 1 - e^{-\lambda(s^2 - t^2)} \tag{12}$$

$$\mathbb{P}(\tau_t > s) = e^{-\lambda(s^2 - t^2)} \tag{13}$$

$$\mathbb{E}[\tau_{t,s}] = \frac{1}{e^{-\lambda t^2} - e^{-\lambda s^2}} \int_t^s u \times 2\lambda u e^{-\lambda u^2} du$$
$$= \frac{1}{e^{-\lambda t^2} - e^{-\lambda s^2}} \left(t e^{-\lambda t^2} - s e^{-\lambda s^2} + \sqrt{\frac{\pi}{\lambda}} [\Phi(\sqrt{2\lambda} s) - \Phi(\sqrt{2\lambda} t)] \right) \tag{14}$$

where Φ is the cumulative distribution function of a standard Gaussian variable.

Lemma 2. *Let $0 \leq t \leq T$, $\lambda > 0$ and r be real numbers. The following holds:*

$$\int_t^T 2\lambda u e^{-ru - \lambda u^2} du = e^{-rt - \lambda t^2} - e^{-rT - \lambda T^2} - r\frac{\sqrt{\pi}}{\sqrt{\lambda}} e^{\frac{r^2}{4\lambda}} \left(\Phi\left(\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right) \tag{15}$$

where Φ is the cumulative distribution function of a standard Gaussian variable.

Proof.

$$\int_t^T 2\lambda u e^{-ru - \lambda u^2} du$$
$$= \int_t^T 2\lambda u e^{-\frac{1}{2}\left(\left[\sqrt{2\lambda} u + \frac{r}{\sqrt{2\lambda}} \right]^2 - \frac{r^2}{2\lambda} \right)} du$$
$$= \int_{\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}}}^{\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}}} \left(z - \frac{r}{\sqrt{2\lambda}} \right) e^{-\frac{1}{2}\left(z^2 - \frac{r^2}{2\lambda} \right)} dz$$
$$= e^{\frac{r^2}{4\lambda}} \left(e^{-\frac{1}{2}\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right)^2} - e^{-\frac{1}{2}\left(\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}} \right)^2} \right) - r\frac{\sqrt{\pi}}{\sqrt{\lambda}} e^{\frac{r^2}{4\lambda}} \left(\Phi\left(\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right)$$
$$= e^{-rt - \lambda t^2} - e^{-rT - \lambda T^2} - r\frac{\sqrt{\pi}}{\sqrt{\lambda}} e^{\frac{r^2}{4\lambda}} \left(\Phi\left(\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right)$$

Lemma 3. *Let $0 \leq t \leq T$ and $s > t$ be constants. Then,*

$$\Theta_{t,T}(r, s) := \mathbb{E}[e^{-r\tau_{t,s}} 1\{\tau_{t,s} \leq T\}]$$
$$= \frac{e^{-rt} - e^{-r(s \wedge T) - \lambda((s \wedge T)^2 - t^2)}}{1 - e^{-\lambda(s^2 - t^2)}}$$
$$- r\frac{\sqrt{\pi}}{\sqrt{\lambda}} \frac{e^{\lambda t^2 + \frac{r^2}{4\lambda}}}{1 - e^{-\lambda(s^2 - t^2)}} \left(\Phi\left(\sqrt{2\lambda}(s \wedge T) + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right) \tag{16}$$

Proof. Using equation (15):

$$\mathbb{E}[e^{-r\tau_{t,s}} 1\{\tau_{t,s} \leq T\}]$$
$$= \frac{1}{e^{-\lambda t^2} - e^{-\lambda s^2}} \int_t^{s \wedge T} 2\lambda u e^{-ru - \lambda u^2} du$$
$$= \frac{1}{e^{-\lambda t^2} - e^{-\lambda s^2}} \left(e^{-rt - \lambda t^2} - e^{-r(s \wedge T) - \lambda(s \wedge T)^2} - r\frac{\sqrt{\pi}}{\sqrt{\lambda}} e^{\frac{r^2}{4\lambda}} \left(\Phi\left(\sqrt{2\lambda}(s \wedge T) + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right) \right)$$

Corollary 4. *Define the particular case $\Theta_{t,T}(r) := \Theta_{t,T}(r, \infty) = \mathbb{E}[e^{-r\tau_t} 1\{\tau_t \leq T\}]$. It is given explicitly by:*

$$\Theta_{t,T}(r) = e^{-rt} - e^{-rT - \lambda(T^2 - t^2)} - r\frac{\sqrt{\pi}}{\sqrt{\lambda}} e^{\frac{r^2}{4\lambda} + \lambda t^2} \left(\Phi\left(\sqrt{2\lambda} T + \frac{r}{\sqrt{2\lambda}} \right) - \Phi\left(\sqrt{2\lambda} t + \frac{r}{\sqrt{2\lambda}} \right) \right) \tag{17}$$

Proof. Take $s = \infty$ into equation (16)

Lemma 5. *Let $0 \le t \le T$, $s > t$ and $D \ge 0$ be constants. Then,*

$$\begin{aligned}\boldsymbol{\Psi}_{t,T}(r, D, s) &:= \mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T} e^{-ru}du\right]\\ &= \frac{\xi_{t,(D+t)\wedge T}(r) - \xi_{t,(D+s)\wedge T}(r)e^{-\lambda(s^2-t^2)}}{1-e^{-\lambda(s^2-t^2)}}\\ &+ \frac{e^{-rD+\lambda t^2+\frac{r^2}{4\lambda}}}{1-e^{-\lambda(s^2-t^2)}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,(s\wedge(T-D)\vee t)+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right)\end{aligned} \tag{18}$$

where

$$\xi_{t,T}(r) := \int_t^T e^{-ru}du = \begin{cases}\frac{e^{-rt}-e^{-rT}}{r} & r \ne 0\\ T-t & r = 0\end{cases} \tag{19}$$

and Φ is the cumulative distribution function of a standard Gaussian variable.

Proof. ***CASE* 1:** $T \le t + D$

$$\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T} e^{-ru}du\right] = \mathbb{E}\left[\int_t^{T} e^{-ru}du\right] = \int_t^T e^{-ru}du = \frac{e^{-rt}-e^{-rT}}{r} \tag{20}$$

CASE 2: $s + D \le T$

In this case,

$$\begin{aligned}&\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T} e^{-ru}du\right]\\ &= \mathbb{E}\left[\int_t^{\tau_{t,s}+D} e^{-ru}du\right]\\ &= \frac{1}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_t^s\left(\int_t^{u+D} e^{-rs}ds\right)2\lambda u e^{-\lambda u^2}du\\ &= \frac{e^{-rt}}{r} - \frac{1}{r}\frac{e^{-rD}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_t^s 2\lambda u e^{-ru-\lambda u^2}du\\ &= \frac{e^{-rt}}{r} - \frac{1}{r}\frac{e^{-r(D+t)-\lambda t^2}-e^{-r(D+t_1)-\lambda s^2}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\\ &+ \frac{e^{-rD+\frac{r^2}{4\lambda}}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,s+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right)\\ &= \frac{\frac{e^{-rt}-e^{-r(D+t)}}{r} - \frac{e^{-rt}-e^{-r(D+s)}}{r}e^{-\lambda(s^2-t^2)}}{1-e^{-\lambda(s^2-t^2)}}\\ &+ \frac{e^{-rD+\lambda t^2+\frac{r^2}{4\lambda}}}{1-e^{-\lambda(s^2-t^2)}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,s+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right)\end{aligned} \tag{21}$$

using equation (15).

CASE 3: $t + D < T < s + D$

In this case,

$$\begin{aligned}&\mathbb{E}\left[\int_{(\tau_{t,s}+D)\wedge T}^{t} e^{-ru}du\right]\\ &= \frac{1}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_t^s\left(\int_t^{(u+D)\wedge T} e^{-rs}ds\right)2\lambda u e^{-\lambda u^2}du\\ &= \frac{1}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_t^{T-D}\left(\int_t^{u+D} e^{-rs}ds\right)2\lambda u e^{-\lambda u^2}du\\ &+ \frac{1}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_{T-D}^{s}\left(\int_t^{T} e^{-rs}ds\right)2\lambda u e^{-\lambda u^2}du\\ &= \frac{e^{-rt}}{r}\frac{e^{-\lambda t^2}-e^{-\lambda(T-D)^2}}{e^{-\lambda t^2}-e^{-\lambda s^2}} - \frac{1}{r}\frac{e^{-rD}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_t^{T-D} 2\lambda u e^{-ru-\lambda u^2}du\\ &+ \frac{e^{-rt}-e^{-rT}}{r}\frac{1}{e^{-\lambda t^2}-e^{-\lambda s^2}}\int_{T-D}^{s} 2\lambda u e^{-\lambda u^2}du\\ &= \frac{e^{-rt}}{r}\frac{e^{-\lambda t^2}-e^{-\lambda(T-D)^2}}{e^{-\lambda t^2}-e^{-\lambda s^2}} - \frac{1}{r}\frac{e^{-r(D+t)-\lambda t^2}-e^{-rT-\lambda(T-D)^2}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\\ &+ \frac{e^{-rD+\frac{r^2}{4\lambda}}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,(T-D)+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right)\\ &+ \frac{e^{-rt}-e^{-rT}}{r}\frac{e^{-\lambda(T-D)^2}-e^{-\lambda s^2}}{e^{-\lambda t^2}-e^{-\lambda s^2}}\\ &= \frac{\frac{e^{-rt}-e^{-r(D+t)}}{r} - \frac{e^{-rt}-e^{-rT}}{r}e^{-\lambda(s^2-t^2)}}{1-e^{-\lambda(s^2-t^2)}}\\ &+ \frac{e^{-rD+\lambda t^2+\frac{r^2}{4\lambda}}}{1-e^{-\lambda(s^2-t^2)}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,(T-D)+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right)\end{aligned} \tag{22}$$

Combining equations (20)–(22), *and adjusting them for the case* $r = 0$ *yields* equation (18).

Corollary 6. *Introduce the shortened notation* $\mathbf{\Psi}_{t,T}(r,\,D)\colon=\mathbf{\Psi}_{t,T}(r,\,D,\,\infty)$. *The following holds*

$$\mathbf{\Psi}_{t,T}(r,\,D)=\mathbb{E}\left[\int_t^{(\tau_t+D)\wedge T}e^{-ru}du\right]$$
$$=\xi_{t,(D+t)\wedge T}(r)+e^{-rD+\lambda t^2+\frac{r^2}{4\lambda}}\frac{\sqrt{\pi}}{\sqrt{\lambda}}\left(\Phi\left(\sqrt{2\lambda}\,(t\vee(T-D))+\frac{r}{\sqrt{2\lambda}}\right)-\Phi\left(\sqrt{2\lambda}\,t+\frac{r}{\sqrt{2\lambda}}\right)\right) \tag{23}$$

where ξ *is given by* equation (19) *and* Φ *is the cumulative distribution function of a standard Gaussian variable.*

Proof. Take $s=\infty$ *in* equation (18).

B. Expected cash-flows formulas

This appendix derives closed-form formulas for the expected value of the mining project. First, recall from equation (5) that $\mathrm{CF}_{t,T}=\mathrm{CF}_{t,T}^{(\lambda)}$ denotes the sum of discounted cash-flows generated by the mine during the time interval $[t,\,T]$, with $0\le t\le T$. We now introduce two additional times t_0 and s with $0\le t_0\le t<s$, and we define $v_{t,T}(t_0,\,S_{t_0},\,s)$ as the expected sum of cash-flows of the mine between t and T, conditionally on the price S_{t_0} (with $t_0\le t$), and conditionally on a tailings dam disaster happening between t and s, with $t<s$ (i.e. $t\le\tau<s$):

$$v_{t,T}\left(t_0,\,S_{t_0},\,s\right)\colon=\mathbb{E}\left[\mathrm{CF}_{t,T}|S_{t_0},\,t\le\tau<s\right] \tag{24}$$

The reason for introducing such a function v is that it is general enough to cover all the required explicit value formulas involving the possibility of a tailings dam disaster. For convenience, we also introduce the notation

$$v_{t,T}^{\mathrm{safe}}\left(t_0,\,S_{t_0}\right)\colon=\mathbb{E}\left[\int_t^T e^{-ru}\Pi(u,\,S_u)du|S_{t_0}\right] \tag{25}$$

corresponding to the expected discounted cash-flows between t and T, conditionally on S_{t_0}, in the case when there is no risk of tailings dam disaster ($\lambda=0$), as well as the shortened notation

$$v_{t,T}\left(t_0,\,S_{t_0}\right)\colon=v_{t,T}\left(t_0,\,S_{t_0},\,\infty\right)=\mathbb{E}\left[\mathrm{CF}_{t,T}|S_{t_0},\,t\le\tau\right] \tag{26}$$

Proposition 7. *The following holds:*

$$v_{t,T}^{\mathrm{safe}}\left(t_0,\,S_{t_0}\right)=qw_S\theta_S\xi_{t,T}(r)+qw_S\left(S_{t_0}-\theta_S\right)e^{\kappa_S t_0}\xi_{t,T}(r+\kappa_S)-qw_C C_0\xi_{t,T}(r-\pi) \tag{27}$$

$$\begin{aligned}&v_{t,T}(t_0,\,S_{t_0},\,s)\\&=qw_S\theta_S(\mathbf{\Psi}_{t,T}(r,\,0,\,s)+\xi_{t,T}(r)-\mathbf{\Psi}_{t,T}(r,\,D,\,s))\\&+qw_S(S_{t_0}-\theta_S)e^{\kappa_S t_0}(\mathbf{\Psi}_{t,T}(r+\kappa_S,\,0,\,s)+\xi_{t,T}(r+\kappa_S)-\mathbf{\Psi}_{t,T}(r+\kappa_S,\,D,\,s))\\&-qw_C C_0(\mathbf{\Psi}_{t,T}(r-\pi,\,0,\,s)+\xi_{t,T}(r-\pi)-\mathbf{\Psi}_{t,T}(r-\pi,\,D,\,s))-\Theta_{t,T}(r-p,\,s)\end{aligned} \tag{28}$$

where $\xi_{t,T}(r)$ *is given by* equation (19), $\Theta_{t,T}(r,\,s)$ *is given by* Lemma 3, *and* $\mathbf{\Psi}_{t,T}(r,\,D,\,s)$ *is given by* Lemma 5.

Proof. First,

$$\begin{aligned}&v_{t,T}^{\mathrm{safe}}(t_0,\,S_{t_0})=\mathbb{E}\left[\int_t^T e^{-ru}\Pi(u,\,S_u)du|S_{t_0}\right]\\&=\int_t^T(e^{-ru}qw_S[\theta_S+(S_{t_0}-\theta_S)e^{-\kappa_S(u-t_0)}]-e^{-ru}qw_C C_0e^{\pi u})du\\&=qw_S\theta_S\int_t^T e^{-ru}du+qw_S(S_{t_0}-\theta_S)e^{\kappa_S t_0}\int_t^T e^{-(r+\kappa_S)u}du-qw_C C_0\int_t^T e^{-(r-\pi)u}du\\&=qw_S\theta_S\xi_{t,T}(r)+qw_S(S_{t_0}-\theta_S)e^{\kappa_S t_0}\xi_{t,T}(r+\kappa_S)-qw_C C_0\xi_{t,T}(r-\pi)\end{aligned} \tag{29}$$

where $\xi_{t,T}(r)=\frac{e^{-rt}-e^{-rT}}{r}1\{r\neq0\}+(T-t)1\{r=0\}$ (equation (19)). *Similarly,*

$$\begin{aligned}&\mathbb{E}\left[\int_t^{(\tau+D)\wedge T}e^{-ru}\Pi(u,\,S_u)du|S_{t_0},\,t\le\tau<s\right]=\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T}e^{-ru}\Pi(u,\,S_u)du|S_{t_0}\right]\\&=\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T}(e^{-ru}qw_S[\theta_S+(S_{t_0}-\theta_S)e^{-\kappa_S(u-t_0)}]-e^{-ru}qw_C C_0e^{\pi u})du|S_{t_0}\right]\\&=qw_S\theta_S\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T}e^{-ru}du\right]+qw_S(S_{t_0}-\theta_S)e^{\kappa_S t_0}\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T}e^{-(r+\kappa_S)u}du\right]\\&-qw_C C_0\mathbb{E}\left[\int_t^{(\tau_{t,s}+D)\wedge T}e^{-(r-\pi)u}du\right]\\&=qw_S\theta_S\Psi_{t,T}(r,\,D,\,s)+qw_S(S_{t_0}-\theta_S)e^{\kappa_S t_0}\Psi_{t,T}(r+\kappa_S,\,D,\,s)-qw_C C_0\Psi_{t,T}(r-\pi,\,D,\,s)\end{aligned} \tag{30}$$

where Ψ *is given by* equation (18). *Finally, using* Lemma 3:

$$\begin{aligned}&\mathbb{E}[e^{-r\tau}P_\tau 1\{\tau\le T\}|t\le\tau<s]=\mathbb{E}[e^{-r\tau_{t,s}}P_{\tau_{t,s}}1\{\tau_{t,s}\le T\}]\\&\quad=\mathbb{E}[e^{-(r-p)\tau_{t,s}}1\{\tau_{t,s}\le T\}]=\Theta_{t,T}(r-p,\,s)\end{aligned} \tag{31}$$

Using equations (30), (29), (31), *and the fact that*

$$\begin{aligned}&\mathbb{E}\left[\int_{(\tau+D)\wedge T}^T e^{-ru}\Pi(u,\,S_u)du|S_t,\,t\le\tau<s\right]\\&=\mathbb{E}\left[\int_{t_n}^T e^{-ru}\Pi(u,\,S_u)du|S_t,\,t\le\tau<s\right]-\mathbb{E}\left[\int_{t_n}^{(\tau+D)\wedge T}e^{-ru}\Pi(u,\,S_u)du|S_t,\,t\le\tau<s\right]\end{aligned}$$

yields equation (28).

C. Stable approximations

The fact that λ is very small in practice can make equations (16) and (18) numerically unstable. This short subsection suggests some simple approximations to deal with this problem.

C.1 Small λ asymptotics

As $\frac{r}{\sqrt{2\lambda}} \to \text{sign}(r)\times\infty$ when $\lambda\to 0$, one can use the asymptotic approximations $\Phi(x) \underset{x\to\infty}{\simeq} 1-\frac{1}{\sqrt{2\pi}x}e^{-x^2/2}$ and $\Phi(x) \underset{x\to-\infty}{\simeq} -\frac{1}{\sqrt{2\pi}x}e^{-x^2/2}$ to obtain the following approximations:

$$\Theta_{t,T}(r) \underset{\lambda\to 0}{\simeq} \frac{2\lambda t}{r}e^{-rt} - \frac{2\lambda T}{r}e^{-rT-\lambda(T^2-t^2)} \tag{32}$$

$$\Psi_{t,T}(r,D) \underset{\lambda\to 0}{\simeq} \xi_{t,T}(r)+ \frac{2\lambda}{r}\left(\frac{(t\vee(T-D))e^{-rT}}{r+2\lambda(t\vee(T-D))} - \frac{te^{-r((t+D)\wedge T)}}{r+2\lambda t}\right) + \frac{e^{-rT}}{r+2\lambda(t\vee(T-D))}(1-e^{-\lambda((t\vee(T-D))^2-t^2)}) \tag{33}$$

We make use of the stable approximations (32) and (33) in our numerical experiments whenever necessary.

C.2 Small |s − t| approximation

When $s\to t$, $\tau_{t,s}$ converges a. s. to t. In the situation when s is close to t, it can be convenient to approximate $\tau_{t,s}$ by $\mathbb{E}[\tau_{t,s}]$, and similarly to approximate $\Theta_{t,T}(r,s)$ by $e^{-r\mathbb{E}[\tau_{t,s}]}1\{\mathbb{E}[\tau_{t,s}]\le T\}$ (see equation (16)) and $\Psi_{t,T}(r,D,s)$ by $\xi_{t,(\mathbb{E}[\tau_{t,s}]+D)\wedge T}(r)$ (see equation (18)), where $\mathbb{E}[\tau_{t,s}]$ is given by equation (14).

D. Quantization-and-interpolation algorithm

For comprehensiveness, we explicitly detail in this Appendix the quantization-and-interpolation algorithm described in Balata et al. (2019), adapted to our context. We use this algorithm for the numerical implementation of the option equations (9) and (11).

D.1 Inverse Gamma process

Before doing so, we need to recall some results regarding the price process $dS_t = \kappa_S(\theta_S - S_t)dt + \sigma_S S_t dW_t$ (equation (2)).

From Zhao (2009), we know that a stationary distribution exists for this process (provided that $\kappa_S + \sigma_S^2/2 > 0$, which is satisfied in our case as $\kappa_S = 0.65 > 0$) and that it is explicitly given by an inverse-gamma distribution $\text{IGa}^{\alpha,\beta}$ with shape parameter $\alpha = 1 + \frac{2\kappa_S}{\sigma_S^2}$ and scale parameter $\beta = \frac{2\kappa_S\theta_S}{\sigma_S^2}$. This result will be useful to build discrete state grids in the next subsection.

In order to solve the dynamic programs (9) and (11) numerically, we need a time-discretization of the price process (2). Langrené et al. (2016) provide an accurate discretization scheme for (2), based on the strong solution for (2) available for example in Zhao (2009), which in particular preserves positivity:

$$S_{t_{n+1}} = F_n(S_{t_n}, G) \tag{34}$$

$$F_n(s,g) := se^{-\delta_n(g)} + \kappa_S\theta_S\frac{1-e^{-\delta_n(g)}}{\delta_n(g)}(t_{n+1}-t_n) \tag{35}$$

$$\delta_n(g) := (\kappa_S + \sigma_S^2/2)(t_{n+1}-t_n) - \sigma_S\sqrt{t_{n+1}-t_n}\,g \tag{36}$$

with $t_0 = 0 < t_1 < t_2 < \ldots < t_N = T$, fixed initial price $S_{t_0} = S_0$, and G is a standard Gaussian variable $\mathcal{N}(0,1)$.

D.2 Price grid

We first build a fixed, sorted price grid $\mathcal{S} := \{s_1, s_2, \ldots, s_M\}$ of size M, for example the following uniform grid:

$$s_m := q_{1/(2M)} + \frac{m-1}{M-1}(q_{1-1/(2M)} - q_{1/(2M)}), \quad m = 1, \ldots, M \tag{37}$$

where $q_p = q_p(\text{IGa}^{\alpha,\beta})$ is the p-quantile of the stationary price distribution $\text{IGa}^{\alpha,\beta}$. With such a price grid construction, the price interval $[s_1, s_M]$ covers the central $(1-1/M)$% of the stationary price distribution. In practice we use $M = 100$ points.

D.3 Quantization

For each price $s\in\mathcal{S}$, we approximate the conditional random variable $S_{t_{n+1}}|S_{t_n} = s$ by quantization. The discrete dynamics (34)-(35)-(36) involves a standard Gaussian variable G. Optimal quantization is one approach to approximate G by a discrete random variable $G^Q := \{g_q, p_q\}_{q=1,\ldots,Q}$ which takes the value g_q with probability p_q. We refer to Pagès et al. (2004) for further information about optimal quantization and how to compute the grid G^Q. In the Gaussian case, precomputed multivariate optimal grids are available on the website http://www.quantize.maths-fi.com. Fig. 7 shows the univariate optimal grid with $Q = 50$ points, which is the one we use in practice.

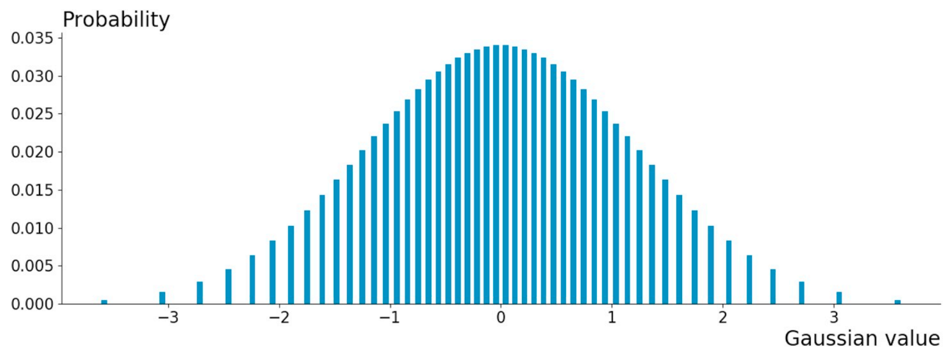

Fig. 7. Optimal univariate Gaussian quantization with $Q = 50$ points

D.4 Dynamic programming

We are now ready to solve the dynamic programs (9) and (11). Without loss of generality, we describe the algorithm in the case of the retrofit option (11). We compute $v^{\text{dry}}(t_n, \mathrm{s}_m)$ for each price value s_m in the grid $\mathscr{S}$, for each time t_n, $n = N, N-1, \dots, 0$.

First, at the final time $t_N = T$, $v^{\text{dry}}(t_N, \mathrm{s}_m) = 0$ for every $m = 1, \dots, M$.

For $0 \leq n < N$, we proceed by induction. Assume that the option value grid

$$\{(\mathrm{s}_1, v^{\text{dry}}(t_{n+1}, \mathrm{s}_1)), \dots, (\mathrm{s}_M, v^{\text{dry}}(t_{n+1}, \mathrm{s}_M))\} \tag{38}$$

has been computed previously. We want to compute the subsequent option value grid

$$\{(\mathrm{s}_1, v^{\text{dry}}(t_n, \mathrm{s}_1)), \dots, (\mathrm{s}_M, v^{\text{dry}}(t_n, \mathrm{s}_M))\} \tag{39}$$

In equation (11), every single term is explicit except for the conditional expectation $\mathbb{E}[v^{\text{dry}}(t_{n+1}, S_{t_{n+1}})|S_{t_n} = \mathrm{s}_m]$. The idea of quantization is to approximate it by

$$\mathbb{E}\left[v^{\text{dry}}\left(t_{n+1}, S_{t_{n+1}}\right)|S_{t_n} = \mathrm{s}_m\right] \simeq \sum_{q=1}^{Q} p_q v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q)) \tag{40}$$

where the Gaussian variable G in the discrete dynamics (34)-(35)-(36) is approximated by the quantization grid $G^Q := \{g_q, p_q\}_{q=1,\dots,Q}$.

At this stage, equation (40) cannot be implemented because we only computed $v^{\text{dry}}(t_{n+1}, \mathrm{s}_m)$ on the price grid points s_m, while there is no guarantee that the subsequent prices $F_n(\mathrm{s}_m, g_q)$ belong to the price grid $\mathscr{S}$. To overcome this problem, we simply estimate $v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q))$ by linear interpolation or extrapolation based on the grid (38) computed at time t_{n+1}:

- If there exists an index l such that $F_n(\mathrm{s}_m, g_q) \in [\mathrm{s}_l, \mathrm{s}_{l+1}]$, we simply estimate $v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q))$ by linear interpolation between the values $v^{\text{dry}}(t_{n+1}, \mathrm{s}_l)$ and $v^{\text{dry}}(t_{n+1}, \mathrm{s}_{l+1})$.
- If $F_n(\mathrm{s}_m, g_q)$ lies outside the price range $[\mathrm{s}_1, \mathrm{s}_M]$, we simply estimate $v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q))$ by linear extrapolation

Denote by Interp() this linear interpolation/extrapolation based on the previously computed value grid (38). We replace the quantization-based conditional expectation approximation (40) by

$$\mathbb{E}\left[v^{\text{dry}}\left(t_{n+1}, S_{t_{n+1}}\right)|S_{t_n} = \mathrm{s}_m\right] \simeq \sum_{q=1}^{Q} p_q \,\mathrm{Interp}(v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q)))$$

D.5 Summary

Algorithm 1 summarizes the whole algorithm to implement the retrofit option equation (11).

Algorithm 1. Quantization-and-Interpolation

Initializations:

1. Define a time grid $t_0 = 0 < t_1 < t_2 < \dots < t_N = T$.
2. Define a price grid $\mathscr{S} := \{\mathrm{s}_1, \mathrm{s}_2, \dots, \mathrm{s}_M\}$ (for example using equation (37)).
3. Define a Gaussian quantization grid $G^Q := \{g_q, p_q\}_{q=1,\dots,Q}$ (for example downloaded from http://www.quantize.maths-fi.com).
4. Set $v^{\text{dry}}(t_N, \mathrm{s}_m) = 0$ for every $m = 1, \dots, M$.

Backward induction:

1. For $n = N-1, \dots, 0$:
   a) Approximate $\mathbb{E}[v^{\text{dry}}(t_{n+1}, S_{t_{n+1}})|S_{t_n} = \mathrm{s}_m]$ by

$$\hat{E}_{n,m} := \sum_{q=1}^{Q} p_q \,\mathrm{Interp}(v^{\text{dry}}(t_{n+1}, F_n(\mathrm{s}_m, g_q)))$$

where the linear interpolation/extrapolation operator Interp() is based on the grid

$\{(s_1, v^{dry}(t_{n+1}, s_1)), ..., (s_M, v^{dry}(t_{n+1}, s_M))\}$

computed during the previous iteration $n + 1$.

b). Implement equation (11) using the approximation $\hat{E}_{n,m}$:

$$v^{dry}(t_n, s_m) = \max\Big\{-e^{-rt_n}C_{dry} + v^{safe}_{(t_n+D_{dry})\wedge T,T}(t_n, s_m),$$

$$v_{t_n,T}(t_n, s_m, t_{n+1})\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1}) + (v^{safe}_{t_n,t_{n+1}}(t_n, s_m) + \hat{E}_{n,m})\mathbb{P}(\tau_{t_n} \ge t_{n+1})\Big\} \quad (41)$$

using the explicit formulas for $v^{safe}_{t_n,t_{n+1}}(t_n, s_m)$ and $v^{safe}_{(t_n+D_{dry})\wedge T,T}(t_n, s_m)$ (equation (27)), $v_{t_n,T}(t_n, s_m, t_{n+1})$ (equation (28)), $\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1})$ (equation (12)) and $\mathbb{P}(\tau_{t_n} \ge t_{n+1})$ (equation (13)).

2. The initial option value $v^{dry}(t_0, S_0)$ can be interpolated from the grid $\{(s_1, v^{dry}(t_0, s_1)), ..., (s_M, v^{dry}(t_0, s_M))\}$

Remark: A byproduct of the maximization (41) is the set of optimal decisions

$$\alpha_{n,m} := 1\Big\{-e^{-rt_n}C_{dry} + v^{safe}_{(t_n+D_{dry})\wedge T,T}(t_n, s_m) >$$

$$v_{t_n,T}(t_n, s_m, t_{n+1})\mathbb{P}(t_n \le \tau_{t_n} < t_{n+1}) + (v^{safe}_{t_n,t_{n+1}}(t_n, s_m) + \hat{E}_{n,m})\mathbb{P}(\tau_{t_n} \ge t_{n+1})\Big\}$$

which are plotted on Fig. 6 (see also Chen et al. (2015) on how to plot optimal decisions).

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